

Generating Artificial Plant Morphologies for Function and Aesthetics through Evolving L-Systems

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Abstract

Due to the replacement of natural flora and fauna with urban environments, a significant part of the earth's organisms that function as primary consumers have been dispelled. To compensate for the reduction in the amount of primary consumers, robotic systems that mimic plant-like organisms are interesting to mimic for their potential functional and aesthetic value in urban environments. To investigate how to utilize plant developmental strategies in order to engender urban artificial plants, we built a simple evolutionary model that applies an L-System based grammar as an abstraction of plant development. In the presented experiments, phytomorphologies (plant morphologies) are iteratively constructed using a context sensitive L-System. The genomic representation of the L-System is subject to mutation by an evolutionary algorithm. These mutations thus alter the developmental rules of these phytomorphologies. We compare the differences between the light absorption of evolving virtual plants that remain static during their life and virtual plants that possess the possibility to move joints that link the separate parts of the virtual plants. Our results show that our evolutionary algorithm did not exploit potential beneficial joint actuation, instead, mostly static structures evolved. The results of our evolving L-System show that it is able to create various phytomorphologies, albeit that the results are preliminary and will be more thoroughly investigated in the future.

Introduction

The development of phytomorphological elements of plants ultimately arose from a dynamic interaction between genetic, ontogenetic and environmental forces. Phytomorphological traits have emerged through the evolution and selection of plants, favoring those that were adequately adapted to their environment. Different environments stimulate the development and evolution of specific qualities in plants and contribute to the adaptation of plants to specific environmental niches. Light-absorption is one of the most essential characteristic prevalent in almost all plants. The resulting role of plants as primary consumers conveys their fundamental impact on any terrestrial ecosystem. Urban environments have replaced a large share of plant-rich environments meaning that the potential energy up-take in these environments is exposed and primed for solar exploitation. For an efficient,

but still aesthetically pleasing, deployment of solar cells, we investigate the developmental processes manifested by years of plant evolution. Hence, we are interested in gaining insights into how plant development works and how this can be mimicked in intelligent robotic and autonomous systems. In order to investigate how to properly embody such systems, an evolutionary developmental simulation model was created for investigating various factors that have contributed to the evolution of phytomorphologies. In the context of *flora-robotica*, a 4 year project funded under the 'EU-Horizon 2020 Future and Emerging Technologies Proactive Action', the developmental methodology for creating artificial plants and eventually robotic and autonomous systems, is developed to investigate how these systems may emerge from the simulated evolution of developmental systems of virtual plants.

Various signaling mechanisms have evolved to communicate environmental factors to remote cells and tissues. Moreover, the cell walls of plant cells contribute to the relative immobility as well as the rigidity of plants, limiting cell migration and actuation. Lacking a nervous system, plants are forced to utilize signaling molecules for communication. These molecules atone for the lack in efficient communication mechanisms through various diffusion and transduction pathways. The signaling molecules can be transported through an apoplastic (through the cell wall) or symplastic (via the cytoplasm; through plasmodesmata) pathway. Various molecules can also be transported over long distances through the vasculature of the plant. Although plants acquired efficient dynamic behavior that directly influences morphogenesis, we are interested in seeing whether phytomorphologies can emerge from simpler abstractions. Since actual robotic implementations of evolved phytomorphologies are likely not able to grow or move once created, grammars seem to be a suitable method to implement. Conventionally, development through local cell communication (or tissue communication) can be simulated by simple grammars (Lindenmayer and Jürgensen, 1992) while more complex communication can be mimicked by implementing morphogens (Wolpert, 1969). Morphogens seem to be more

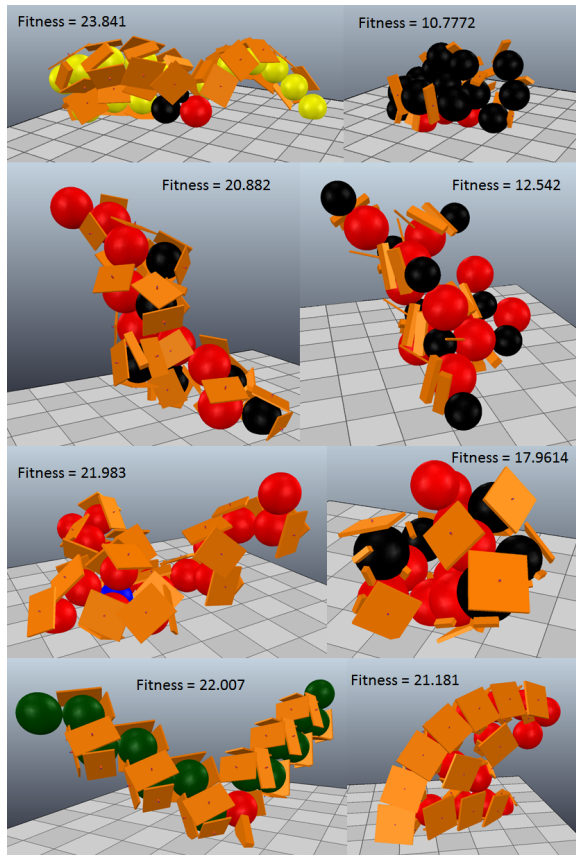


Figure 1: Eight of the best evolved static individuals with their respective fitness values.

relevant abstractions taken from biology than L-Systems though L-Systems are easier to implement. Moreover, since L-Systems work with variables, they can potentially be extended to contain signal propagation algorithms and even morphogens themselves. In this paper, L-Systems are implemented to engender the phytomorphogenesis of artificial plants with the aim of evaluating the possible generation of physical systems that act as primary energy consumers in urban environments.

Background

Phytomorphogenesis

Variation in plant features is influenced by many factors including ecophysiological, phenological, morphological and ontological traits. Other important factors driving plant-evolution include resource allocation, biochemistry, metabolism, and leaf morphology and function (Ackery et al., 2000). All the genes are in turn subject to evolution and specific genetic components are selected across generations. The absolute fitness value of a plant is roughly determined by the amount of viable seeds it produces during its lifetime that is promoted by the previously mentioned at-

tributes.

It has been shown that the photosynthetic rate of leaves in plants has a direct influence on the absolute fitness of in *arabidopsis thaliana*. One specific gene (Alt1g61800) causes leaves to produce more chloroplasts when plants were placed in a different environment where they were subjected to a higher light-intensity (Athanasious et al., 2010) demonstrating the importance of dynamic feedback for plants. However, solar cells are less affected by environmental factors such as temperature and do not necessarily have to rely on complex feedback processes to function optimally. Since dynamic behavior in plants is usually a result of various compromises taken to optimize for survival and reproduction, we conceive that plants grown in controlled conditions do not have to rely on dynamic feedback as much. We therefore focus on investigating the more intrinsic properties of plants that contribute to the static generation of phytomorphologies within a degree of stochasticity.

Phyllotaxis is the main factor driving phytomorphogenesis (Cells, 1997). The most common patterns formed in plants through phyllotaxis include distichous, spiral, decusate and whorled patterns (Kuhlemeier, 2007). Notably, the divergence angles of primordia of the plants differ usually by 180° , 90° , 137.5° (Newell and Shipman, 2005) and some other more uncommon angles (Kuhlemeier, 2007). These, mostly unimodal, angles influence how well the leaves sprouting from the primordia can absorb light and overshadow other leaves (Falster and Westoby, 2003). Leaves can also be positioned at a certain level of steepness which is advantageous for either preventing self-shading or capturing light from the morning and evening sun (Falster and Westoby, 2003). Steeper angles of lamina are also more beneficial for plants that receive an amount of light higher than the maximum photosynthetic potential of a plant. When the leaves are steeply oriented, other leaves, that would otherwise be overshadowed, can receive more light and thus the overall photosynthetic activity of the plant is increased. Other evolutionary trade-offs that emerge in the leaves of plants include e.g. mass-to-area ratio, sap flow versus heat processing, CO₂ uptake to water loss ratio and the leaf size-to-number ratio (Nicotra et al., 2011). Moreover, hormones, such as auxin, play an important role in embryonic development, cellular elongation and phyllotaxis (Prasad and Dhonukshe, 2013). Despite the importance of these driving factors for the development of plants, these factors would greatly convolute the evolutionary search space.

Simulated models

Computer models of plants have generally been implemented in computer graphics (Habel et al., 2009), for accurate modeling of plant dynamics (Runions et al., 2014; Cournède et al., 2008; Merks and Guravage, 2013; Prusinkiewicz and Runions, 2012) and for assessing the role of evolution on the emergence of plant traits (Valladares and

Pearcy, 2000). Moreover, evolutionary computations and generative encodings have been implemented to efficiently simulate plant models (Zamuda and Brest, 2012, 2014) with some biological accuracy. In previous work on generating patterned morphologies, and for keeping the morphological encoding simple, generative encoding strategies, such as the parametric encoding used in the work of Sims (Sims, 1994), are usually implemented since they can recursively generate body segments. Different types of generative encoding strategies have been developed over the past two decades to abstract developmental strategies towards generating both morphology and control of virtual creatures (Eggenberger-Hotz, 1997; Yeom and Park, 2010). One strategy for generating artificial structures linked to neural networks is known as artificial ontogeny (Bongard and Pfeifer, 2001, 2003). In this method, an agents simulated spherical elements can grow by increasing in size and splitting in two. As a result, repeated divisions can transit a single unit in a fully developed agent. Each separately created unit contained up to six joints and diffusion sites. These diffusion sites could in turn contain zero or more sensory, motor and interneurons. Despite a promising application of artificial ontogeny to produce plant-like structures, the implementation of neural networks can result in a great increase of the search-space making it a less attractive system to implement for our current purposes.

A Lindenmayer system (L-system) is another grammatical generative encoding approach, originally used to mimic plant development by iteratively rewriting variables and constants through a set of rules (Lindenmayer, 1968; Prusinkiewicz, 1997). L-Systems can be seen as a developmental representation of a virtual plant. comparable to other generative encoding strategies, the similarity of L-Systems to Biology includes their modularity of the reuse of rules and variables comparable to how organisms reuse genes. Further relevance L-systems have to biology can be derived from the fact that cells, or parts of plants, can change their state, or cell fate. This determines the behavior and ultimately the phytomorphogenesis of plant form and structure. L-Systems are thus an attractive method to implement for our purposes both as they somewhat mimic biological development as well as being simple and efficiently encoded. L-systems have furthermore been used to create the morphological structure of virtual creatures with reactive controllers (Hornby and Pollack, 2001). This approach can similarly be effective for the generation of virtual plants.

Methodology

Virtual Robot Experimentation Platform (V-REP) (Rohmer et al., 2013) is used as the simulator to create and evaluate plant-like robotic morphologies. The simulated components are controlled via a C++ based DLL plugin created with visual studio 2013. The plugin is divided into three parts: a genetic algorithm, a morphology generator and a control part.

The genome of the morphology is encoded as the rules and parameters of the L-Systems. Two experiments were done to simulate 16 evolutionary runs for evolving static plant-like morphologies as well as 16 runs for evolving plant-like morphologies in which joints could rotate.

Genetic Algorithm

The implemented genetic algorithm is a steady state genetic algorithm (Wu and Chow, 1995). And in our case, a random offspring is generated asexually, without crossover, and evaluated against a random individual in the population. The random selection and a population size of 100 individuals was used to keep the population somewhat diverse and to slower the convergence of the evolving L-System to a local optima. The genomes of the initial population were furthermore randomly initialized. The individuals were evaluated based on their ability to absorb light in an environment that only contained a flat surface, a light-source and the individual itself. When comparing an evaluated offspring with a random individual of the population, the offspring would only replace the selected individual if its fitness value was higher. Based on preliminary experiments, the mutation rate was set to 5% meaning that each variable of the genome had a 5% chance of being changed. When mutating the variables, either a completely new random value could be assigned to the specific variable, or a local mutation could cause the value to change locally. These local mutations are most effective to explore the local search-space of a population of individuals.

Ten evaluation steps contribute to the eventual fitness value of a virtual plant. At each time-step, the amount of light absorbed by the simulated leaves of an individual is calculated. The orientation and surface area of the leaves have a direct influence on the amount of light absorbed by the leaves. The amount of light absorbed is calculated by the multiplication of one light-sensitive surface area of the artificial leaf with the z-directional vector of the leaf relative to the directional vector that is oriented from the leaf's origin to the origin of the light-source. Furthermore, if there is anything between the artificial leaf and the light-source, the leaf will does not contribute to the fitness value of the individual. The light-source that directly influences the fitness of the virtual plants is moved at each time-step. Starting at the Cartesian coordinate (2.0,-4.0,10.0) and ending at the coordinate (2.0,5.0,10.0). The sun thus moves in the direction of y with a directional vector of (0.0, 1.0, 0.0) as illustrated in figure 3.

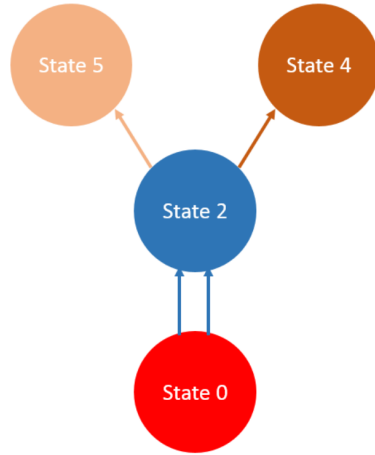
The fitness function for each individual is given in equation 1. The fitness F is the sum of the acquired fitness values after ten time-steps i . n represents the total amount of evaluation time-steps. The total amount of leaves is given by o , and p represents the total amount of objects formed by the individual being evaluated. A represents the surface area of the artificial leaves which is multiplied by the z directional

L-System Genotype

States: 0,1,2,3,4,5
 Constants: [,], +
 Axiom: 0
 State Production Rules:
 0 → 0+[2]+[2]
 2 → 2+[4]+[5]

Example of two iterations:
 1st:
 0 → 0+[2]+[2]
 2nd:
 0+[2]+[2] →
 0+[2+[4]+[5]]+[2+[4]+[5]]

Symbolic representation



Phenotype

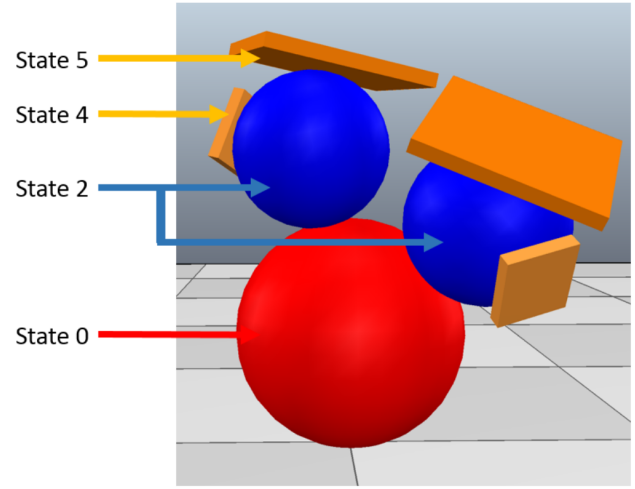


Figure 2: Three illustrations of the implemented L-System are shown. The Genotypic representation shows how the production rules result in the generation of the morphology. The symbolic representation shows the developmental instructions and the relationships between states as similarly represented by the work of Sims (Sims, 1994). The phenotype generated by the example is shown on the right. Note that the + constant represents a three dimensional orientation to which a new object is rotated relative to its parent.

vector θ . V represents the volume of the objects.

$$F = \sum_{i=1}^n \left(\sum_{j=1}^o S_{ij} \theta_{ij} - \sum_{k=1}^p V_{ik} \right) \quad (1)$$

L-System

The implemented L-System was a context sensitive L-system. In our case, the context refers mainly to the simulated environment. For example, in order to prevent objects from overlapping, a feedback loop to the L-System ensures that the created morphology does not contain any overlapping/colliding objects. The L-system contains a total of 10 variables which are referred to as specific states of the objects that are created. Each state of the object contains corresponding rule sets that define what child objects are created. An example of how the states, rules and constants of the L-System influence morphogenesis is displayed in figure 2.

The L-System generates morphologies by iterating seven times through the state parameters of the morphology. Seven iterations were subjectively chosen as they seemed to exhibit a good diversity of morphologies without requiring too much computational power. The axiom of the L-System is a state 0 object. Before the first iteration of the L-System, an object in state 0 is therefore created at the center of the environment on top of the floor. Afterwards, the first iteration of the L-system will generate objects that the rules in state 0 produce. Having only seven iterations, an object chain from the initial object to the outer most child consists of a

maximum of 8 objects. Some loopholes in the L-System can quickly result in a very high computational demand and thus specific constraints are implemented. Every object in a given state can potentially create up to six new child objects. The maximum amount of objects that can be created is therefore limited to 50. Likewise, the amount of loops the L-System can make for generating these objects is limited to 200. To enable individuals to absorb light from the environment, two object states of the L-System genome represent artificial leaves that are expressed as rectangular cuboids. These leaves are colored orange. All other states represent spherical objects that shape the overall morphology. Spherical objects were chosen in order to effortlessly calculate the position of new objects without having to worry about collisions and overlapping objects. The objects in four other object states are colored red, blue, green and yellow while the remaining objects are colored black by default. Note that the first object created is always in state zero which is always colored red. An illustration of how the L-System generates the phenotype from a specific genome is depicted in figure 2.

Additional parameters are included in the L-System to enable movement of the joints. Whether a joint moves is represented by one Boolean. The angular rotation a joint can make per time-step is limited to 36 degrees meaning that a joint can rotate a maximum of 360 degrees in a positive or negative direction during one evaluation.

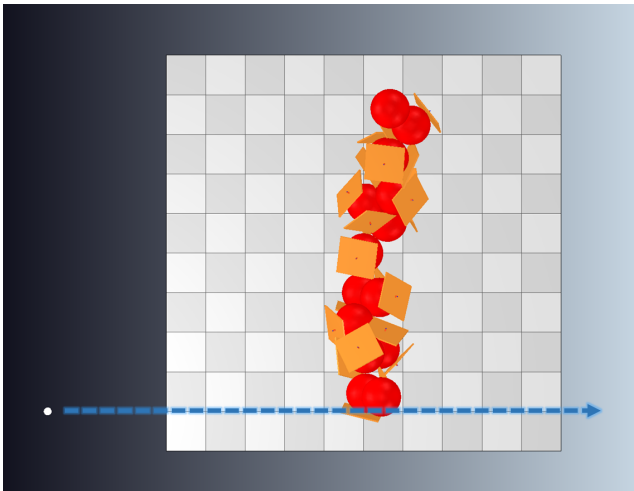


Figure 3: This figure shows the top-down view of the simulation environment with an omnidirectional light-source shown as a white dot in the bottom left corner. The dashed blue line represents the movement of the light-source.

Results

As can be seen in figure 5, the average acquired fitness values of the population with static virtual plants is similar to the fitness of the population of plants that could potentially actuate their joints. Since the evolutionary runs were not normally distributed (confirmed by a Shapiro Wilk test) a Mann-Whitney U Test was performed to see whether the results were significantly different. The Mann-Whitney U test confirms that the data is insufficient to reject its null hypothesis as can also be inferred by looking at the graph (figure 5). No statistical difference between the efficiency of static versus actuated phenotypes could be seen for the amount of simulated generations. A difference might emerge when simulating far more generations considering that the runs shown in figure 5 did not plateau. Although a few phenotypes did utilize moving parts (such as in figure 7), the majority of the phenotypes that evolved did not move. In the 16 evolutionary runs of rotating individuals, the best individuals of the final generations seldom utilized any actuation in joints that would change the shape of the artificial plants significantly.

Although the fitness values depicted in the graph of figure 5 seem quite arbitrary, they can be explained with some additional information. For example, the fitness value of the best evolved individual (figure 6) was 23.841. Without the negative contribution of the volume of the individual, its fitness would have been 31.939. The division of this value by the amount of time steps results in the average surface area of the artificial leaves that was exposed to the light-source. This area is corrected by the relative angle the leaves had in respect to the light-source. 3.194 m^2 is thus the 2D-projection of the average light absorbing surface area of the

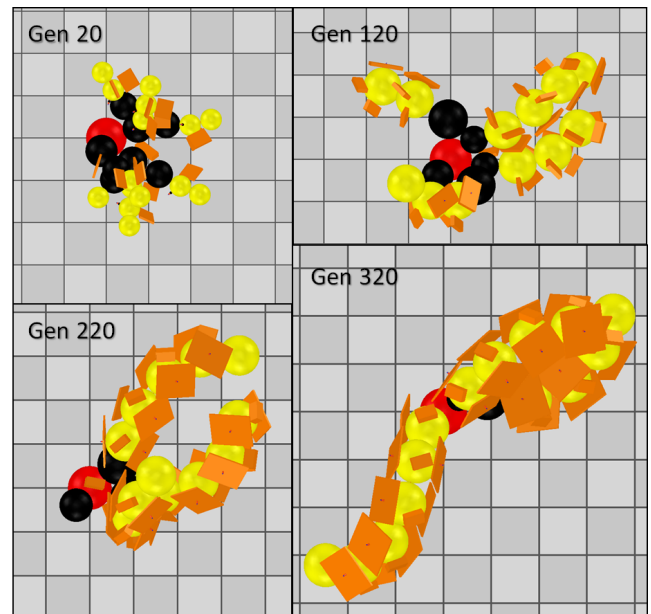


Figure 4: A top view of individuals of one evolutionary run are depicted to illustrate how evolution shapes new more efficient individuals.

artificial leaves. The total volume of an individual could also be extracted by checking the negative contribution of the volume. In the given example, the total negative fitness contribution of the volume of the individual discussed in this paragraph was 8.099. The total volume of the simulated individual was thus 0.8099 m^3 . Hence, the phenotype seen in figure 6 represents a structure with an average light-absorption area of 3.19399 m^2 and a volume of 0.80989 m^3 .

The phenotypes of the evolved phytomorphologies are quite diverse and different spiral patterned morphologies can be seen (figure 1). In figure 4, the best evolutionary run is mapped across different generations. Looking at the top view of this figure, one can see that the total amount of surface area exposed by the artificial leaves (orange rectangles) gradually becomes larger.

Discussion

In this paper, we aimed to see how an evolutionary developmental algorithm can engender various phytomorphologies optimized to absorb light. As can be seen in figure 1, a wide variety of phytomorphologies evolved. Functionally, these evolved morphologies don't look particularly optimal for light absorption as one would expect all the orange surfaces to point somewhat upwards instead of in the various directions shown in the resulting morphologies. Making longer evolutionary runs could shed more light on whether the evolutionary L-System can actually generate more efficient models. Actuating the morphologies did not change the population fitness values significantly when compared to the

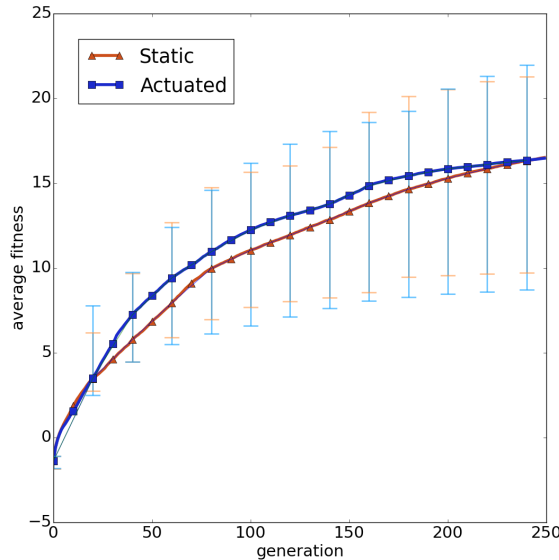


Figure 5: The figure depicts the average fitness values of the populations across generations. The runs are not significantly different from one another (p-value was 0.782) when using the Mann-Whitney U test).

statically simulated populations. Blind tracking of a moving light-source may have caused the search space to become more convoluted making the algorithm inept for finding solutions where actuation was more beneficial than not actuating anything.

The evolved virtual plants were quite voluminous considering that the volume has a negative effect on the fitness value. However, making large objects and dispersing the morphology over a large area, while making leaves with a thin volume but large surface area, is an intuitive result given the simulation environment. It is expected that different phytomorphologies arise when artificial plants have an

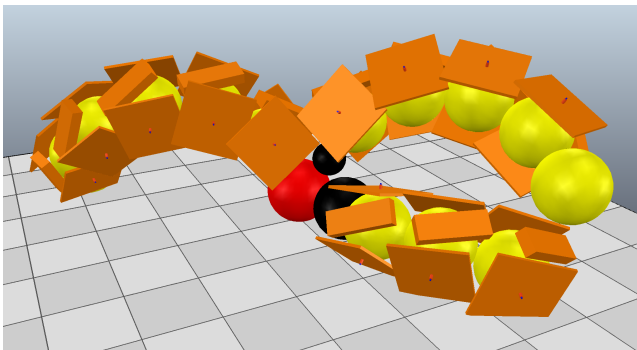


Figure 6: The phenotype of the best evolved individual. Note that object chains are surrounded by artificial leaves

additional restriction to grow horizontally. In biological environments, factors such as the overshadowing of neighboring plants cause additional pressure that stimulate specific types of plants to grow tall quickly. Co-evolving the same L-System can therefore yield results that are more diverse than the ones presented in this paper.

Considering the results, various future improvements of the genetic algorithm may increase the efficiency of a population to traverse the search space. Implementing a crossover function might definitely increase the efficiency of the evolving L-System considering that specific states and rules of the L-System can be recombined between individuals within the population to make better performing offspring. As mentioned earlier, the implementation of neural networks in addition to artificial development (as done by Bongard (Bongard and Pfeifer, 2003)) can be interesting for developing more dynamic morphologies. Morphogens (Wolpert, 1969) are also an attractive strategy to implement in order to mimic long range communication in plants. An algorithm that checks for diversity besides quality, as has been implemented in novelty search (Lehman and Stanley, 2008) might also be useful to speed up the search process. Moreover, novelty search can lead to the evolution of very distinct morphologies making it more useful for people that would like to generate phytomorphological structures for aesthetic purposes.

Conclusion

We have shown that our evolving L-System can create various phytomorphologies that are evolved to maximize light absorption. These phytomorphologies were generated to consider implementing them in urban environments for both functional and aesthetic motives. Evolution did not exploit possibly beneficial joint actuation but rather converged on various types of static phytomorphologies instead. In the future, this evolving L-System can be extended by implementing additional algorithms to increase the effectiveness of traversing the state space landscape for acquiring both more efficient and more unique phytomorphologies.

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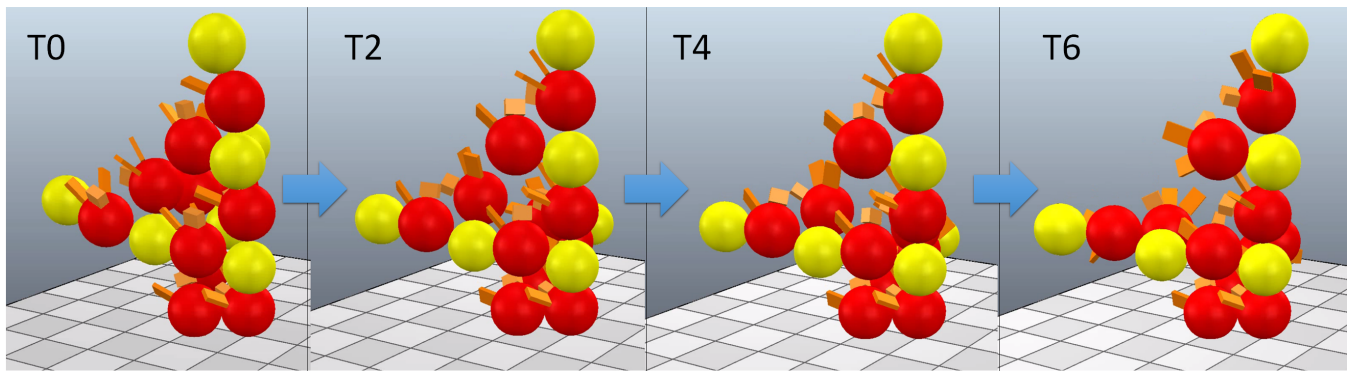


Figure 7: An individual that rotated some of its joints during the simulation. T0, T2, T4 and T6 represent the respective time steps 0, 2, 4 and 6

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